COT 3100C Homework #1

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1 Question 1

a).

From the 3-13 mile mark:

$$D = R \cdot T$$
$$D_J = D_A = 10\, miles$$
$$D_J = 10 = R_J \cdot T_J$$
$$R_J = \frac{10}{T_J} \frac{miles}{hour}$$
$$D_A = 10 = (R_J - 2\, \frac{miles}{hour})(T_J + \frac{25}{60}\, hours)$$
$$D_A = 10 = (\frac{10}{T_J} - 2)(T_J - \frac{25}{60})$$
$$10 = 10 + \frac{250}{60T_J} - 2T_J - \frac{50}{60}$$
$$0 = \frac{250}{60T_J} - 2T_J - \frac{50}{60}$$
$$-\frac{250}{60T_J} = -2T_J - \frac{50}{60}$$
$$-250 = -120T_J^2 - 50T_J$$
$$-120T_J^2 - 50T_J + 250 = 0$$
$$T_J = \frac{50 \pm \sqrt{50^2 - 4(-120)(250)}}{2(-120)} = -\frac{5}{3}, \frac{5}{4}$$
$$T_J > 0,\ T_J = \frac{5}{4}\, hours$$
$$D_J = 10 = R_J \cdot \frac{5}{4}$$
$$R_J = 8\, \frac{miles}{hour}$$

From the 0-3 mile mark:

$$R_J = 8\, \frac{miles}{hour} = R_A$$

Thus, both started running at 8 $\frac{miles}{hour}$ at the beginning of the race.

b).

From the 0-13 mile mark:

$$D = R \cdot T$$

$$D_J = 13\, miles = 8\, \frac{miles}{hour} \cdot T_J$$

$$T_J = \frac{13}{8} = 1.625\, hours$$

$$((((1.625\, hours - 1\, hour) \cdot 60) - 37\, minutes) \cdot 60) - 30\, seconds = 0$$

Thus, the time elasped for Joanna was 1:37:30.

c).

From the 0-13 mile mark:

$$T_A = \frac{13}{8} + \frac{25}{60} = \frac{49}{24} \approx 2.041667\, hours$$

$$((((2.041667\, hours - 2\, hours) \cdot 60) - 2\, minutes) \cdot 60) - 30\, seconds \approx 0$$

Thus, the time elasped for Ahmed was 2:02:30.

2 Question 2

$$\log_2(x^2) + \log_4(y) = 7$$

$$\log_2(x^2) + \frac{\log_2(y)}{\log_2(4)} = 7$$

$$\log_2(x^2) + \frac{\log_2(y)}{2} = 7$$

$$2\log_2(x) + \frac{\log_2(y)}{2} = 7$$

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$$\log_4(8x) + \log_2(2y^2) = 8$$

$$\frac{\log_2(8x)}{\log_2(4)} + \log_2(2y^2) = 8$$

$$\frac{\log_2(8x)}{2} + \log_2(2y^2) = 8$$

$$\frac{\log_2(x)}{2} + \frac{\log_2 8}{2} + \log_2(y^2) + \log_2 2 = 8$$

$$\frac{\log_2(x)}{2} + \frac{3}{2} + \log_2(y^2) + 1 = 8$$

$$\frac{\log_2(x)}{2} + \log_2(y^2) = \frac{11}{2}$$

$$\frac{\log_2(x)}{2} + 2\log_2(y) = \frac{11}{2}$$

Let variable a = $\log_2 x$, and let variable $b = \log_2 y$.

$$2\log_2(x) + \frac{\log_2(y)}{2} = 7$$

$$\frac{\log_2(x)}{2} + 2\log_2(y) = \frac{11}{2}$$

$$2a + \frac{1}{2}b = 7$$

$$\frac{1}{2}a + 2b = \frac{11}{2}$$

$$\frac{1}{2}a = -2b + \frac{11}{2}$$

$$a = -4b + 11$$

$$2(-4b + 11) + \frac{1}{2}b = 7$$

$$-8b + 22 + \frac{1}{2}b = 7$$

$$-\frac{15}{2}b = -15$$

$$b = 2$$

$$a = -4(2) + \frac{11}{2} = 3$$

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$$a = 3 = \log_2(x)$$
$$x = 2^3 = 8$$
$$b = 2 = \log_2(y)$$
$$y = 2^2 = 4$$

Thus, the two equations intersect at (8,4).

3 Question 3

$\neg(p \land q) \lor \neg(\neg r \oplus s)$

p	q	$(p \land q)$	r	s	$(\neg r \oplus s)$	$\neg(\neg r \oplus s)$	$\neg(p \land q)$	$\neg(p \land q) \lor \neg(\neg r \oplus s)$
F	F	F	F	F	T	F	T	T
F	F	F	F	T	F	T	T	T
F	F	F	T	F	F	T	T	T
F	F	F	T	T	T	F	T	T
F	T	F	F	F	T	F	T	T
F	T	F	F	T	F	T	T	T
F	T	F	T	F	F	T	T	T
F	T	F	T	T	T	F	T	T
T	F	F	F	F	T	F	T	T
T	F	F	F	T	F	T	T	T
T	F	F	T	F	F	T	T	T
T	F	F	T	T	T	F	T	T
T	T	T	F	F	T	F	F	F
T	T	T	F	T	F	T	F	T
T	T	T	T	F	F	T	F	T
T	T	T	T	T	T	F	F	F

4 Question 4

Prove $(\neg(p \land q) \implies \neg q) \Leftrightarrow (p \lor \neg q)$ with a truth table.

p	q	$(p \land q)$	$\neg(p \land q)$	$\neg q$	$(\neg(p \land q) \implies \neg q)$	$(p \lor \neg q)$
F	F	F	T	T	T	T
F	T	F	T	F	F	F
T	F	F	T	T	T	T
T	T	T	F	F	T	T

Thus, the statement $(\neg(p \land q) \implies \neg q) \Leftrightarrow (p \lor \neg q)$ is a tautology.

5 Question 5

Prove $(\neg(p \land q) \implies \neg q) \Leftrightarrow (p \lor \neg q)$ with laws of logic.

#	Statement	Reason
1	$(\neg(p \land q) \implies \neg q)$	Given
2	$q \implies (p \land q)$	Contrapositive
3	$\neg q \lor (p \land q)$	Implication Identity
4	$(\neg q \lor p) \land (\neg q \lor q)$	Distributive Law
5	$(\neg q \lor p) \land (q \lor \neg q)$	Commutative Law
6	$(\neg q \lor p) \land T$	Inverse Law
7	$\neg q \lor p$	Identity Law
8	$p \lor \neg q$	Commutative Law

Thus, $(\neg(p \land q) \implies \neg q) \Leftrightarrow (p \lor \neg q)$.

6 Question 6

Prove the following argument with the rules of inference.

$$p \implies r$$

$$s$$

$$\neg q$$

$$s \implies (p \lor q)$$

$$-------$$

$$\therefore r$$

#	Statement	Reason
1	s	Given
2	$s \implies (p \lor q)$	Given
3	$(p \lor q)$	Modus Pones w/ #1, #2
4	$\neg q$	Given
5	p	Disjunctive Syllogism w/ #3, #4
6	$p \implies r$	Given
7	$\therefore r$	Modus Pones w/ #5, #6

7 Question 7

Prove the following argument with the rules of inference.

$$p \implies t$$

$$\neg u$$

$$\neg r \lor s$$

$$\neg q \implies \neg t$$

$$u \lor (p \lor r)$$

$$-------$$

$$\therefore q \lor s$$

#	Statement	Reason
1	$u \lor (p \lor r)$	Given
2	$\neg u$	Given
3	$p \lor r$	Disjunctive Syllogism w/ #1, #2
4	$p \implies t$	Given
5	$\neg q \implies \neg t$	Given
6	$t \implies q$	Contrapositive w/ #5
7	$p \implies q$	Law of Syllogism w/ #4, #6
8	$\neg p \lor q$	Implication Identity w/ #7
9	$r \lor q$	Rule of Resolution w/ #3, #8
10	$\neg r \lor s$	Given
11	$\therefore q \lor s$	Rule of Resolution w/ #9, #10

8 Question 8

Prove/disprove the following claim:

$$\forall x[\exists y[xy - x - y = -1]]$$

This statement is true for all x when y = 1. We can prove this by first simplifying the expression:

$$xy - x - y = -1$$

$$xy - y = x - 1$$

$$y(x - 1) = x - 1$$

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$$\frac{y(x-1)}{(x-1)} = \frac{(x-1)}{(x-1)}$$
$$y = 1$$

For all x from $(-\infty, \infty)$, y is equal to 1.

Plug in 1 for y in the expression $xy - x - y = -1$ and simplify:

$$x(1) - x - (1) = -1$$
$$x - x - 1 = -1$$
$$-1 = -1$$

This statement is always true. Thus, for all x when y = 1, the claim $xy-x-y = -1$ is true.

9 Question 9

Gerolamo Cardano was a Renaissance mathematician who lived from 1501 to 1576. His father was a mathematician who was friends with the famous artist Leonardo da Vinci. He was one of the pioneers and key figures of probability and statistics. His book *Liber de ludo aleae* described probability and how to create favorable odds for gambling. He was also one of the first to write about the the binomial theorem and bionomial coefficents, important topics in statistics in the book *Opus novum de proportionibus*.

Cardano was also heavily influencial in alegbra. He created a solution to the cubic equation $ax^3 + bx + c = 0$ in his book *Ars magna*. He also knew of the existance of imaginary numbers, although he did not understand how they worked. He also had works in geometry, particually in hypocycloids. In his later years, he practiced medicine until his death in 1576 at the age of 74.

Sources:
1. `https://en.wikipedia.org/wiki/Gerolamo_Cardano`
2. `https://www.britannica.com/biography/Girolamo-Cardano`